Application of surface acoustic waves to direct measurement of the parameters of superfluid helium thin films

1990 J. Phys.: Condens. Matter 2 5047

(http://iopscience.iop.org/0953-8984/2/22/023)

J. Phys.: Condens. Matter 2 (1990) 5047–5052. Printed in the UK

Application of surface acoustic waves to direct measurement of the parameters of superfluid helium thin films

Yu A Kosevich† and E S Syrkin‡
† All-Union Surface and Vacuum Research Centre, Moscow 117334, USSR
‡ Institute for Low Temperature Physics and Engineering, Ukrainian SSR Academy of Sciences, Kharkov 31064, USSR

Received 19 April 1989, in final form 15 January 1990

Abstract. We propose an acoustic method to measure directly the density of the normal component in thin films of superfluid helium. The effect of the helium film on the velocity and the penetration depth of Love and Gulyaev–Bleustein shear surface waves is analysed. The advantage of the method based on the application of shear surface waves is that the change in their velocity, as is shown, is proportional to the normal component density and the helium film thickness.

1. Introduction

Among the problems of the physics of liquid helium is that of the study of its superfluid properties in thin films, and in particular the measurement of the superfluid and normal component densities [1]. The superfluid component in a thin film can be investigated by well developed methods based on measurement of the velocity of third sound and attenuation in the superfluid film itself [1] on precise measurement of the change in frequencies of torsional [2] and standing acoustic vibrations [3–5] of a resonator interacting with the helium film. Recently, however, there have emerged more efficient methods of measurement of the properties of liquids (or gases) in contact with solids; these employ acoustic sensors based on the properties of running acoustic waves [6, 7]. They measure the surface mass, caused by a thin adsorbed layer with an accuracy of up to 10^{-10} g cm^{-2} [7]; This value corresponds to an incomplete monolayer of atoms. Application of surface acoustic waves to measurement of the parameters (the density ρ_n and the viscosity η) of the normal component of superfluid He II can be based on the fact that acoustic vibrations of a solid substrate in its plane generate the vibrations only of the normal component of He II. Therefore the He II layer causes the small changes in the velocity and attenuation of a surface acoustic wave in a solid substrate.

In this paper we discuss the method of measurement of the He II normal component density in a thin film, based on running shear surface waves (SSWs), i.e. purely transverse acoustic waves polarised in the boundary plane.§ The influence of ρ_n on the properties of SSWs of various natures is investigated: in a solid layer–substrate system (Love waves

§ The idea of this method has been briefly described in [8].

[9]) and at specific sections of piezoelectric crystals (Gulyaev–Bleustein [10, 11] waves). Measurement of the He II normal component based on the ssws has the advantage over that based on Rayleigh surface waves because, in ssws, only the normal component moves, while, in Rayleigh waves which involve vertical displacement of the solid boundary, both He II components move. It is worth mentioning that, since in non-superfluid liquids one has $\rho_r = \rho$, then from the temperature dependence of the change of the ssw velocity it is also possible to determine the superfluid transition point and the density $\rho_s = \rho - \rho_n$ of the superfluid component in He II thin films.

2. A film of He II on an elastic half-space

Let us consider elastic vibrations of the boundary of an isotropic solid in its plane in the presence of a He II layer of thickness h upon it. The equations of motion in such a system reduce to those of elasticity theory for a solid and those for a viscous wave in a He II layer:

$$\rho_n \dot{v}_{ny} = \eta \Delta v_{ny} \tag{1}$$

where v_n is the normal component velocity and the Oy axis lies in the plane of the boundary and is perpendicular to the wave propagation direction (the Ox axis), the Oz axis being along the outer normal to the boundary. The boundary conditions at the interface $z = 0$ between the solid and the liquid He II are

$$v_{ny} = \dot{u}_y \qquad \sigma_{zy} = \eta\, \partial v_{ny}/\partial z \tag{2}$$

where $\boldsymbol{u}$ is the vector of the solid boundary elastic displacement, $\sigma_{zy} = C_{44}\, \partial u_y/\partial z$ is the elastic stress tensor and C_{44} is the elastic shear modulus. On the free surface ($z = h$) the boundary condition is

$$\eta\, \partial v_y/\partial z = 0. \tag{3}$$

Equation (1) and the boundary conditions (2) and (3) result in the following dispersion equation for the shear surface wave due to the presence of the He II layer:

$$C_{44}\sqrt{k^2 - D\omega^2/C_{44}} = \mathrm{i}\omega\eta\sqrt{k^2 - \mathrm{i}\rho_n\omega/\eta}\, \tanh(\sqrt{k^2 - \mathrm{i}\rho_n\omega/\eta}\, h) \tag{4}$$

where ω is the wave frequency, k is the wavenumber and D is the solid density; v_{ny}, $u_y \sim \exp(\mathrm{i}kx - \mathrm{i}\omega t)$.

We shall consider the case realised in ultrasonic experiments, where the shear elastic wave velocity is much larger than the viscous wave velocity:

$$k^2 \ll \rho_n \omega/\eta$$

i.e.

$$\omega \ll \rho_n C_{44}/D\eta. \tag{5}$$

In this case, if the layer thickness exceeds the viscous length of He II, i.e. $h \gg \delta = (2\eta/\rho_n\omega)^{1/2}$, then equation (4) leads to the following expressions for the velocity v and for the inverse penetration depth κ of ssws in a solid:

$$v = C_{44}/\sqrt{C_{44}D + \mathrm{i}\rho_n\omega\eta} \tag{6}$$

$$\kappa \equiv \sqrt{k^2 - D\omega^2/C_{44}} = [(1+\mathrm{i})/\sqrt{2}](\omega/C_{44})\sqrt{\rho_n\omega\eta}. \tag{7}$$

As seen from equations (6) and (7), the parameter of He II influencing the ssw properties is the product $\rho_n\eta$, i.e. to determine the density of the normal component it is necessary to know the viscosity of He II from independent experiments (and vice versa).

Similar SSWs were considered in [12] at the interface between a solid substrate and a non-superfluid ($\rho_n = \rho$) viscous liquid (or gas).

In the case of a He II layer thickness smaller than the viscous depth ($h \ll \delta$) that we are interested in, equation (4) yields the following expression for the inverse penetration depth of the SSWs:

$$\sqrt{k^2 - D\omega^2/C_{44}} = (h\omega/C_{44})(\rho_n\omega + \mathrm{i}\eta k^2) \tag{8}$$

i.e. when $\rho_n\omega \gg \eta k^2$ the properties of SSWs are only affected by the normal component density, unlike the case of bulk He II (where $h \gg \delta$). From equation (8) we find the surface acoustic wave velocity

$$v^2 = C_{44}/D - (\rho_n h\omega/D)^2 \tag{9}$$

i.e. owing to the thin He II layer the wave velocity changes as the square of $\rho_n h$:

$$\Delta v/v = \tfrac{1}{2}(\rho_n h\omega/Dv_0)^2 \tag{10}$$

where $v_0 = (C_{44}/D)^{1/2}$ is the bulk shear wave velocity in the particular direction. For the characteristic parameters of the solid substrate given by $D = 2\ \mathrm{g\,cm^{-3}}$, $v_0 = 3 \times 10^5\ \mathrm{cm\,s^{-1}}$ and $\omega = 2\pi 10^9\ \mathrm{s^{-1}}$ and the thin non-saturated helium film with $\rho_n = 4 \times 10^{-2}\ \mathrm{g\,cm^{-3}}$ for $T = 1.5$ K [13] and $h = 3 \times 10^{-7}$ cm, we obtain $\Delta v/v \simeq 10^{-8}$, which is barely within the sensitivity of modern measurements [7, 14]. The wave velocity changes are so small because the SSWs exist totally as a result of the presence of the He II layer; when $\rho_n h = 0$ we have $\kappa = 0$.

Naturally, in the experiment it is more convenient to measure the parameters of the He II film if the change in the SSW velocity due to the thin layer is linear as a function of $\rho_n h$. This can be so if SSWs exist also in the absence of a He II layer. Such a situation is the case, for example, in a retarding layer–substrate system (Love waves [9]) and also at some interfaces in piezocrystals (Gulyaev–Bleustein [10, 11] waves).

3. A He II film–elastic layer–elastic half-space system

Let a He II film of thickness h be on an elastic layer of thickness d, which is in acoustic contact with an elastic half-space. If the He II film thickness is smaller than the viscous length, then in the low-frequency limit (5) the boundary conditions to the equations of the elasticity theory at the He II–solid interface ($z = d$) are

$$\sigma_{z\alpha} = -\sigma_n \ddot{u}_\alpha \qquad \sigma_n = \rho_n h \tag{11}$$

$$\sigma_{zz} = -\sigma\ddot{u}_z \qquad \sigma = (\rho_n + \rho_s)h \equiv \rho h \tag{12}$$

where the subscripts $\alpha = 1, 2$ indicate the coordinate axes in the tangential plane, the Oz axis is directed along an outer normal to the solid, $\boldsymbol{u}$ is the vector of the solid boundary elastic displacement, and σ_n and σ are excess surface masses due to the normal and total densities of the He II thin film. The boundary conditions (11) and (12) suggest that the vibrations of the solid boundary in its plane moves only the normal component of the He II thin film and its vertical displacements move both components as a single whole. The boundary condition (12) for a He II thin film is the consequence of more general conditions at the boundary of bulk He II with a solid in the case of boundary displacement normal to the surface [15]. These boundary conditions reflect the law of conservation of mass, which is valid for the total density of superfluid He II.

From equations (11) and (12) it also follows that in the case $\ddot{u}_z \ll \ddot{u}_x$ we can neglect the mass loading σ_{zz} caused by the total surface density of the He II thin film. Thus, we can neglect the coupling of the motion of both components of superfluid helium for the SSWs propagating along the solid surface with gently sloping irregularities. For the real solid surface with gently sloping irregularities this conclusion is valid for both $l < h$ and $l > h$, where l is the characteristic root mean square height of surface roughness. (The surface irregularities are gently sloping in the case of small degree of corrugation roughness: $l \ll \xi_{\|}$, where $\xi_{\|}$ is the transverse correlation length of irregularities in the boundary plane (see, e.g., [16]).) The smallness of the characteristic height of surface irregularities compared with the thickness of a He II layer ($l < h$) when the coupling of the motion of both components of superfluid helium is disregarded is really required only in the case of high degree of surface roughness ($l \geqslant \xi_{\|}$).

From the boundary condition (11) at the interface $z = d$ and the usual boundary conditions (2) at the interface $z = 0$ of the elastic half-space and the layer, we obtain the following dispersion equation for a SSW, propagating in such a system:

$$D_0 S_{0\perp}\kappa_0 = DS_\perp^2\kappa_1 \tan(d\kappa_1) + \sigma_n\omega^2[1 + (D_0\kappa_0 S_{0\perp}/DS_\perp^2\kappa_1)\tan(d\kappa_1)]$$
$$\kappa_0 = [(S_{0\|}^2/S_{0\perp}^2)k^2 - \omega^2/S_{0\perp}^2]^{1/2} \qquad \kappa_1 = (\omega^2/S_\perp^2 - k^2 S_\|^2/S_\perp^2)^{1/2} \tag{13}$$

where $S_\|$ and $S_{0\|}$ are the velocities of bulk transverse elastic waves polarised perpendicular to the sagittal plane in the direction parallel to the interface, and $S_\perp$ and $S_{0\perp}$ are the velocities of bulk transverse waves in the direction perpendicular to the boundary (the subscript 0 corresponds to the elastic half-space). The dispersion equation (13) is valid in the general case when in the system of an elastically anisotropic layer at an anisotropic substrate the sagittal plane of the wave coincides with the mirror symmetry plane in both the media. The difference between the velocities of shear waves in directions parallel and perpendicular to the boundary is due to elastic anisotropy of the layer and the substrate. Putting $\sigma_n = 0$ in equation (13), we obtain a dispersion equation for a SSW in the system of an elastic anisotropic layer at an anisotropic substrate [17]; when $d = 0$, $S_\perp = S_\| = S$ and $S_{0\perp} = S_{0\|} = S_0$, we have equation (8) in the low-frequency limit (5). Note that in the absence of the middle layer, when $d = 0$, it is possible by using the boundary conditions (11) and (12) to calculate also the influence of the He II thin film on the Rayleigh surface wave velocity. The change in the Rayleigh wave velocity appears to be linear as a function of the surface mass parameters σ_n and σ, and therefore it is proportional to the He II film thickness. However, contrary to the SSWs velocity, the change in the Rayleigh wave velocity depends on the linear combination of σ_n and σ, and thus on the densities of both the normal and the superfluid components of the He II thin film.

The dispersion equation (13) when $d\kappa_0 \ll 1$ and $d\kappa_1 \ll 1$ has the following form:

$$D_0 S_{0\perp}^2\sqrt{(S_{0\|}^2/S_{0\perp}^2)k^2 - \omega^2/S_{0\perp}^2} = \omega^2[\sigma_n + Dd(1 - S_\|^2/S_{0\|}^2)] > 0. \tag{14}$$

This equation specifies the velocity of the SSW in the case of $S_\|^2/S_{0\|}^2 < 1 + \sigma_n/Dd$. Note in this connection that long-wavelength (low-frequency) SSWs in this system can exist also in the case of an 'accelerating' middle elastic layer ($1 < S_\|^2/S_{0\|}^2 < 1 + \sigma_n/Dd$)—in distinction from the ordinary Love waves in the system of a 'retarding' elastic layer at a substrate [9].

From equation (14) we obtain the expression for the change in the SSW velocity due to the He II thin film for which $\sigma_n \ll Dd/(1 + S_\parallel^2/S_{0\parallel}^2)$ in the case of a retarding middle layer ($S_\parallel < S_{0\parallel}$):

$$\Delta v/v_0 = \sigma_n Dd(1 - S_\parallel^2/S_{0\parallel}^2)\omega^2/S_{0\perp}^2 D_0^2 \qquad v_0 \simeq S_{0\parallel} \tag{15}$$

i.e. it is a linear function of the parameter σ_n and a quadratic function of the frequency ω. For the characteristic parameters of the He II thin film, the retarding layer and the elastic substrate given by $\rho_n \simeq 4 \times 10^{-2}\,\mathrm{g\,cm^{-3}}$, $h \simeq 3 \times 10^{-7}\,\mathrm{cm}$, $\omega \simeq 10^8\,\mathrm{s^{-1}}$, $D \simeq D_0 \simeq 2\,\mathrm{g\,cm^{-3}}$, $d(1 - S_\parallel^2/S_{0\parallel}^2) \simeq 10^{-4}\,\mathrm{cm}$ and $S_{0\perp} \simeq 4 \times 10^5\,\mathrm{cm\,s^{-1}}$, from equation (15) we obtain $\Delta v/v \simeq 10^{-6}$. It means that for the same parameters of the helium film and the elastic half-space as in equation (10), in the considered case the change in the SSW velocity is two orders of magnitude larger even at lower frequencies. We emphasise in this context that equations (14) and (15) are true under the following restrictions on the frequency $\omega : \kappa_1 d \simeq \omega d/S_\perp \ll 1$. It is worth mentioning that, as seen from equation (15), in the case of the elastically anisotropic half-space with $S_{0\perp} < S_{0\parallel}$, the change in the SSW velocity due to the He II thin film increases with decrease in the ratio $S_{0\perp}/S_{0\parallel} \ll 1$, i.e. with increase in the elastic anisotropy.

4. A He II thin film on the surface of a piezocrystal

Owing to the coupling between electric (magnetic) and acoustic fields the Gulyaev–Bleustein SSWs can propagate along the specific sections of piezoelectrics (piezomagnetics) even without the retarding elastic layer (see, e.g., [10, 11, 18–20]). These surface waves have a large penetration depth and as has been shown in [21] their properties are very sensitive to the near-surface distortions of the crystal acoustic parameters, and in particular to the excess surface density. Therefore, the presence of a He II thin film on the piezocrystal surface should affect the SSW velocity. Let us consider the propagation of the Gulyaev–Bleustein SSWs on the (100) plane in the direction [010] of a hexagonal piezocrystal. Proceeding from the mechanical boundary conditions (11) and the conventional quasi-static electric boundary conditions, we obtain the following dispersion equation:

$$\sqrt{k^2 - D\omega^2/\bar{C}_{55}} = k[\mathcal{K}^2/(1 + \varepsilon) + k\sigma_n/D] \tag{16}$$

where $\bar{C}_{55} = C_{55} + 4\pi\beta^2/\varepsilon$ and $\mathcal{K}^2 = 4\pi\beta^2/\bar{C}_{55}\varepsilon$ is the electromechanical coupling parameter of the crystal (β is the piezoelectric modulus and ε is the dielectric permeability of the crystal). When $k\sigma_n/D \ll \mathcal{K}^2$, the relative change in the SSW velocity due to the He II layer is linear as a function of σ_n and ω; it is as follows:

$$\Delta v/v_0 = \mathcal{K}^2\sigma_n\omega/\sqrt{D\bar{C}_{55}} \qquad v_0 = \sqrt{\bar{C}_{55}/D}. \tag{17}$$

With the same parameters of the He II thin film as in above cases, the piezocrystal parameters $D = 2.2\,\mathrm{g\,cm^{-3}}$, $v_0 = 4 \times 10^5\,\mathrm{cm\,s^{-1}}$, $k^2 \simeq 10^{-2}$ (e.g. quartz) and $\omega = 2\pi 10^9\,\mathrm{s^{-1}}$, from equation (17) we obtain $\Delta v/v_0 \simeq 10^{-6}$. This means that the change in the SSW velocity due to the presence of a superfluid He II film of a thickness of the order of tens of the interatomic spacings can be revealed experimentally even in not very strong piezocrystals.

Thus, by measuring the velocity change of a shear surface wave in a crystal, it is possible to determine the density of the normal component of a He II thin film. If a He II film is placed on the surface of a non-piezoelectric crystal, the change in the SSW velocity is a quadratic function of the parameter $\sigma_n = \rho_n h$. In piezocrystals or in the presence of an intermediate retarding layer the change in the SSW velocity is a linear function of the parameter σ_n. The effects discussed demonstrate that modern acoustic methods (acoustic sensors) are adequate to measure the parameters of superfluid helium in thin films, including helium films in porous media [22] and near-monolayer films of ^{3}He–^{4}He superfluid mixtures [23].

References

[1] Putterman S J 1974 *Superfluid Hydrodynamics* (Amsterdam: North-Holland)

[2] Borovikov A P and Peshkov V P 1976 *Zh. Eksp. Teor. Fiz.* **70** 300 (Engl. Transl. 1976 *Sov. Phys.–JETP* **43** 156)

[3] Chester M, Yang L C and Stephens J B 1972 *Phys. Rev. Lett.* **29** 211

[4] Chester M and Yang L C 1973 *Phys. Rev. Lett.* **31** 1377

[5] Lea M J, Spencer D S and Fozooni P 1989 *Phys. Rev.* B **39** 6527

[6] Ricco A S and Martin S J 1987 *Appl. Phys. Lett.* **50** 1474

[7] Martin S J, Ricco A S and Ginby D S 1987 *IEEE Trans. Ultrasonics, Ferroelectrics, Frequency Control* **UFFC-34** 142

[8] Kosevich Yu A 1988 *Proc. 15th All-Union Conference on Low Temperature Physics (Leningrad, 1988)* (Leningrad: Physico-Technical Institute) p 86

[9] Auld B A 1973 *Acoustic Fields and Waves in Solids* vol II (New York: Wiley) p 94

[10] Gulyaev Yu V 1969 *Sov. Phys.–JETP Lett.* **9** 37

[11] Bleustein J L 1968 *Appl. Phys. Lett.* **13** 412

[12] Plesski V P and Ten Yu A 1984 *Zh. Eksp. Teor. Fiz. Pis. Red.* **5** 296 (Engl. Transl. 1984 *Sov. Phys.–Tech. Phys. Lett.* **10** 124)

[13] Esselson V N, Grigoriev V N, Ivantsov V G and Rudavskii E Ya 1978 *Properties of Liquid and Solid Helium* (Moscow: Standartov) (in Russian) p 224

[14] Alers G A 1968 *Physical Acoustics* vol IV, part A (New York: Academic) p 322

[15] Kagan M Yu and Kosevich Yu A 1988 *J. Low Temp. Phys.* **14** 433

[16] Maradudin A A 1978 *Physics of Phonons (Lecture Notes in Physics 285)* ed T Paszkiewicz (Berlin: Springer) p 82

[17] Kosevich Yu A and Syrkin E S 1988 *Sov. Phys.–Acoust.* **34** 61

[18] Farnell G W 1978 *Acoustic Surface Waves* (Berlin: Springer) p 13

[19] Dieulesaint E and Royer D 1980 *Elastic Waves in Solids* (New York: Wiley) p 511

[20] Garcia Del Rio J C and Velasco V R 1985 *Surf. Sci.* **162** 138

[21] Kosevich Yu A and Syrkin E S 1987 *Zh. Eksp. Teor. Fiz. Pis. Red.* **13** 1439 (Engl. Transl. 1987 *Sov. Phys.–Tech. Phys. Lett.* **13** 602)

[22] Mulders N and Beamish J R 1989 *Phys. Rev. Lett.* **62** 438

[23] Wang X and Gasparini F M 1988 *Phys. Rev.* B **38** 11245